Chapter 10
Probability

10.5 Permutations and Combinations

Permutations

Given the letters ABCD,
how many ways can you arrange them?

ABCD	BACD	BCAD	BCDA
ABDC	BADC	BDAC	BDCA
ACBD	CABD	CBAD	CBDA
ACDB	CADB	CDAB	CDBA
ADBC	DABC	DBAC	DBCA
ADCB	DACB	DCAB	DCBA

Here's another method using the Counting Principle

4 ___ 3 ___ 2 ___ 1 ___ = 4! = 24 ways

Counting Principle: If one event can occur in p different ways and another event can occur in q different ways, then there are p * q ways both events can occur.

10.5 Permutations and Combinations

Permutations

Given the letters ABCD,
how many ways can you arrange them if you can reuse letters?

$\underline{\ 4\ }\ \underline{\ 4\ }\ \underline{\ 4\ }\ \underline{\ 4\ }$ = 256 ways

Counting Principle: If one event can occur in p different ways and another event can occur in q different ways, then there are p * q ways both events can occur.

10.5 Permutations and Combinations

Permutations

Given the letters ABCD, how many ways can you arrange them?

4 ___ 3 ___ 2 ___ 1 ___

= 24 ways

Given the letters ABCD, how many ways can you arrange them if you can reuse letters?

4 ___ 4 ___ 4 ___ 4 ___ = 256 ways

Practice -

1. How many different words can you make with the word DAVEGS if you have to use all letters just once?

6! = 720

2. How many 7 digit phone numbers can begin with the prefix 366-XXXX ?

10 * 10 * 10 * 10 = 10,000

10.5 Permutations and Combinations

Factorial

- **Definition** - The product of the integers from 1 to n. (Only positive integers)

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$$

Example: $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

Special case: $0! = 1$

10.5 Permutations and Combinations

Permutations

Given the letters ABCD, how many ways can you arrange them?

$\underline{\;4\;}\;\underline{\;3\;}\;\underline{\;2\;}\;\underline{\;1\;}$

= 24 ways

Given the letters FREE, How many unique ways can you arrange them?

$$\text{Total} = \frac{4!}{?!} = \frac{4!}{2!} = 4 \cdot 3 = 12$$

FREE	RFEE	REFE	REEF
FERE	ERFE	ERFE	EREF
FEER	EFER	EEFR	EERF

Given the letters ESSENTIAL, How many unique ways can you arrange them?

$$\text{Total} = \frac{9!}{?!} = \frac{9!}{2!2!} = 90{,}720$$

10.5 Permutations and Combinations

Permutations

Given the letters FREE,
How many unique ways can you arrange them?

$$\text{Total} = \frac{4!}{?!} = \frac{4!}{2!} = 4 \cdot 3 = 12$$

Given the letters ESSENTIAL,
How many unique ways can you arrange them?

$$\text{Total} = \frac{9!}{?!} = \frac{9!}{2!2!} = 90{,}720$$

Practice -

1. PARALLEL
How many unique ways can you arrange the letters?

$$\frac{8!}{2!3!} = 3{,}360$$

2. Estelle has 8 quarters, 5 dimes, 3 nickels and a penny. How many ways can she arrange them in a line?

$$\frac{17!}{8!5!3!} = 12{,}252{,}240$$

10.5 Permutations and Combinations

Permutations

There are 8 chairs and 5 students.
How many different seating arrangements are possible?

Use the counting principle

8 7 6 5 4

$$= \frac{8!}{(8-5)!} = \frac{8!}{3!}$$

$$= 6720 \ ways$$

Permutation
The number of permutations of n objects taken r at a time.

$$_nP_r = \frac{n!}{(n-r)!}$$

The position of objects matter

10.5 Permutations and Combinations

Permutations

There are 8 chair and 5 students.
How many different seating arrangements are possible?

$$_nP_r = \frac{n!}{(n-r)!} \qquad _8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = 6720 \ ways$$

Practice -

1. Sally has 7 candles, each of a different color. How many ways can she arrange the candles in a candelabra that holds 3 candles?

210

2. Kevin has 12 different CD's, but has a CD case that can hold only 8. How many ways can he arrange the CD's in the case?

19,958,400

10.5 Permutations and Combinations

Permutations

There are 8 chair and 5 students.
How many different seating arrangements are possible?

$$_nP_r = \frac{n!}{(n-r)!} \qquad _8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = 6720 \ ways$$

Practice -

1. Find the value of $_{a+2}P_2$

$$(a+2)(a+1)$$

10.5 Permutations and Combinations

Multiplication, Addition & Complement Principles

How many license plates can be made by using 2 letters followed by 3 digits?

26 26 10 10 10 = 676,000

Practice -

1. The password to an online email account is alpha-numeric. There must be exactly 4 characters/#'s. How many passwords are possible?

1,679,616

2. You have seven different scrabble tiles. How many possible 4 letter words can you make?

840

10.5 Permutations and Combinations

Multiplication, Addition & Complement Principles

How many 3 digit numbers have at least one 7?

7	(9)	(9)		7	7	(9)		7	7	7		
(8)	7	(9)	225 ways	(8)	7	7	26 ways				1 way	
(8)	(9)	7		7	(9)	7						

= 252 ways

A better way: All 3 digit #'s minus all 3 digit #'s without a 7

900 - (8)(9)(9) = 252 ways

10.5 Permutations and Combinations

Multiplication, Addition & Complement Principles

How many 3 digit numbers have at least one 7?

7	(9)	(9)		7	7	(9)		7	7	7	
(8)	7	(9)	225 ways	(8)	7	7	26 ways				1 way
(8)	(9)	7		7	(9)	7					

= 252 ways

Practice -

1. How many 4 digit #'s contain at least one 8 or 9?
Hint:
(all possible)-(#'s w/o 8 or 9)

5,416

2. How many 4-digit #'s contain 2 or more zeros?

252

10.5 Permutations and Combinations

Combinations

- **Definition** - An arrangement of objects in which order is NOT important.

Example 1: The combinations of 2 letters in the word JULY.

6 ways - JU, JL, JY, UL, UY, LY

Example 2: The combinations of all the letters in the word JULY.

only 1 way - JULY

10.5 Permutations and Combinations

Combinations Formula

The number of combinations of n objects taken r at a time, where $r \leq n$, is given by:

$$ {}_nC_r = \frac{n!}{r!(n-r)!} $$

Order does not matter!!!!

Example 1: The combinations of 2 letters in the word JULY.

$$ {}_4C_2 = \frac{4!}{2!(4-2)!} = 6 \textit{ ways} $$

Example 2: The combinations of all the letters in the word JULY.

$$ {}_4C_4 = \frac{4!}{4!(4-4)!} = 1 \textit{ way} $$

10.5 Permutations and Combinations

Combinations Formula

The number of combinations of n objects taken r at a time, where $r \leq n$, is given by:

$$_{n}C_{r} = \frac{n!}{r!(n-r)!}$$

Order does not matter!!!!

Practice

1. You order a sandwich at a restaurant. You can choose 2 side dishes from a list of 8. How many combinations of side dishes are possible?

$$_{8}C_{2} = \frac{8!}{2!(8-2)!} = 28$$

10.5 Permutations and Combinations

Permutations

There are 8 chair and 5 students.
How many different seating arrangements are possible?

$$Total = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = \frac{8!}{3!} = \frac{8!}{(8-5)!}$$

$$= 6{,}720 \; ways$$

$${}_nP_r = \frac{n!}{(n-r)!}$$

Order of objects matter

Combinations

There are 8 players and 5 are chosen to start on the team.
How many combinations can be formed?

$$Total = \frac{8!}{(8-5)!(5)!} = 56 \; ways$$

Order doesn't matter

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

10.5 Permutations and Combinations

Combinations

There are 8 players and 5 are chosen to start on the team. How many combinations can be formed?

$$Total = \frac{8!}{(8-5)!(5)!} = 56\ ways$$

Order doesn't matter

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

Practice -

1. From a list of 12 books, how many groups of 5 can be selected?

792

2. How many baseball teams of 9 can be formed from 14 players?

2,002

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